

Consistency of $\Lambda\Lambda$ hypernuclear events

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Abstract Highlights of $\Lambda\Lambda$ emulsion events are briefly reviewed. Given three accepted events, shell-model predictions based on p -shell Λ hypernuclear spectroscopic studies are shown to reproduce $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{10}\text{Be})$ and $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{13}\text{B})$ in terms of $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He})$. Predictions for other species offer judgement on several alternative assignments of the ${}_{\Lambda\Lambda}^{13}\text{B}$ KEK-E176 event, and on the assignments ${}_{\Lambda\Lambda}^{11}\text{Be}$ and ${}_{\Lambda\Lambda}^{12}\text{Be}$ suggested recently for the KEK-E373 HIDA event. The predictions of the shell model, spanning a wide range of A values, are compared with those of cluster models, where the latter are available.

Keywords hypernuclei · shell model · cluster models

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1 Introduction

$\Lambda\Lambda$ hypernuclei provide valuable information on the $\Lambda\Lambda$ interaction and how it fits into our understanding of the baryon-baryon interaction. Although the existence of $\Lambda\Lambda$ hypernuclei nearly rules out a stable H dibaryon, a ΞN dominated H resonance might affect the systematics of $\Lambda\Lambda$ binding energies. Only three emulsion events presented serious candidates for $\Lambda\Lambda$ hypernuclei before 2001: ${}_{\Lambda\Lambda}^{10}\text{Be}$ [1, 2], ${}_{\Lambda\Lambda}^6\text{He}$ [3] and ${}_{\Lambda\Lambda}^{13}\text{B}$ [4, 5]. The $\Lambda\Lambda$ binding energies $B_{\Lambda\Lambda}$ deduced from these events indicated that the 1S_0 interaction $V_{\Lambda\Lambda}$ was strongly attractive, with a $\Lambda\Lambda$ excess binding energy $\Delta B_{\Lambda\Lambda} \sim 4.5$ MeV, although it had

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been realized that the binding energies of ${}_{\Lambda\Lambda}^{10}\text{Be}$ and ${}_{\Lambda\Lambda}^6\text{He}$ were inconsistent with each other [6]. Here, the $\Lambda\Lambda$ excess binding energy is defined as

$$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) - 2\bar{B}_\Lambda({}_{\Lambda}^{(A-1)}Z), \quad (1)$$

where \bar{B}_Λ is the $(2J+1)$ -average of B_Λ values for the ${}_{\Lambda}^{(A-1)}Z$ hypernuclear core levels. For comparison, $\Delta B_{\Lambda N}({}_{\Lambda}^5\text{He}) = 1.73 \pm 0.13$ MeV, implying the unnatural ordering $\Delta B_{\Lambda\Lambda} > \Delta B_{\Lambda N}$. This perception changed in 2001 when a uniquely assigned ${}_{\Lambda\Lambda}^6\text{He}$ hybrid-emulsion event [7], with updated values [8]

$$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) = 6.91 \pm 0.16 \text{ MeV}, \quad \Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) = 0.67 \pm 0.17 \text{ MeV}, \quad (2)$$

ruled out the high value of $\Delta B_{\Lambda\Lambda}$ from the dubious earlier ${}_{\Lambda\Lambda}^6\text{He}$ event [3], restoring thus the expected hierarchy $\Delta B_{\Lambda\Lambda} < \Delta B_{\Lambda N}$. Both capture at rest formation $\Xi^- + {}^{12}\text{C} \rightarrow {}_{\Lambda\Lambda}^6\text{He} + t + \alpha$ and weak decay ${}_{\Lambda\Lambda}^6\text{He} \rightarrow {}_{\Lambda}^5\text{He} + p + \pi^-$, in this so called NAGARA event, yield consistently with each other the values listed in (2). Neither ${}_{\Lambda\Lambda}^6\text{He}$ nor ${}_{\Lambda}^5\text{He}$ have excited states that could bias the determination of $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He})$.

Accepting the NAGARA event calibration of $V_{\Lambda\Lambda}$, we review and discuss (i) particle stability for lighter $\Lambda\Lambda$ hypernuclei; (ii) reinterpretation of the events assigned ${}_{\Lambda\Lambda}^{10}\text{Be}$ and ${}_{\Lambda\Lambda}^{13}\text{B}$; (iii) several alternative assignments for the ${}_{\Lambda\Lambda}^{13}\text{B}$ event; and (iv) plausibility of the assignments ${}_{\Lambda\Lambda}^{11}\text{Be}$ or ${}_{\Lambda\Lambda}^{12}\text{Be}$ proposed for the recently reported HIDA event [8]. In the course of doing so, we compare $B_{\Lambda\Lambda}$ values derived from emulsion events with shell-model predictions [9] and with selected few-body cluster calculations [6, 10, 11] where the latter exist.

2 Onset of $\Lambda\Lambda$ hypernuclear stability

From the very beginning it was recognized that $\Lambda\Lambda$ and $\Lambda\Lambda N$ were unbound [12, 13]; if $\Lambda\Lambda N$ were bound, the existence of a $nn\Lambda$ bound state would follow. The existence of a ${}_{\Lambda\Lambda}^4\text{H}$ bound state was claimed by AGS-E906 [14], from correlated weak-decay pions emitted sequentially by $\Lambda\Lambda$ hypernuclei produced in a (K^-, K^+) reaction on ${}^9\text{Be}$. However, the ${}_{\Lambda\Lambda}^4\text{H}$ interpretation is controversial [15, 16]. Several post-2001 calculations exist for ${}_{\Lambda\Lambda}^4\text{H}$. A Faddeev-Yakubovsky 4-body calculation finds no bound state [17], whereas a stochastic-variational (SV) 4-body calculation finds it to be bound by as much as 0.4 MeV [18]. The more comprehensive s -shell Λ - and $\Lambda\Lambda$ -hypernuclear SV calculation of Ref. [19] finds ${}_{\Lambda\Lambda}^4\text{H}$ to be particle stable by as little as a few keV, which would be insufficient to maintain particle stability once $V_{\Lambda\Lambda}$ is renormalized to reproduce the recently updated (smaller) value of Eq. (2) for $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He})$.

Regardless of whether ${}_{\Lambda\Lambda}^4\text{H}$ is particle-stable or not, there is a general consensus that the mirror $\Lambda\Lambda$ hypernuclei ${}_{\Lambda\Lambda}^5\text{H}$ – ${}_{\Lambda\Lambda}^5\text{He}$ are particle-stable, with $\Delta B_{\Lambda\Lambda} \sim 0.5 - 1$ MeV [20, 21], or larger owing to the $\Lambda\Lambda - \Xi N$ coupling which is particularly effective here [19, 22, 23]. In addition, substantial charge symmetry breaking effects are expected in these systems, resulting in a higher binding energy of ${}_{\Lambda\Lambda}^5\text{He}$ by up to 0.5 MeV with respect to ${}_{\Lambda\Lambda}^5\text{H}$ [23, 24]. Figure 1

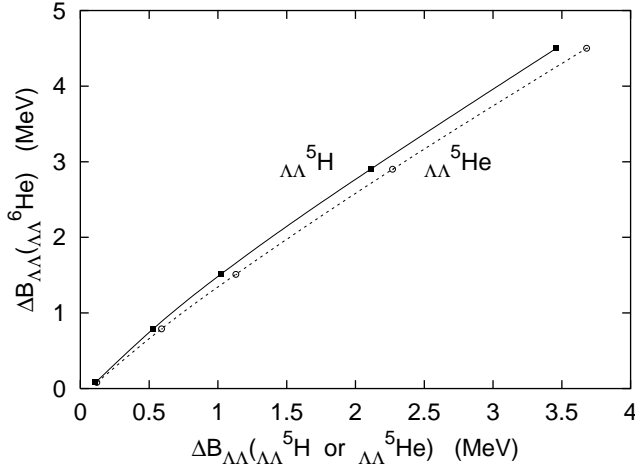


Fig. 1 Faddeev calculations of $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda^6\text{He})$ vs. $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda^5\text{H}, \Lambda\Lambda^5\text{He})$ [20], see text.

demonstrates how $\Delta B_{\Lambda\Lambda}$ values for the $A = 5, 6$ systems, calculated over a broad range of $V_{\Lambda\Lambda}$ strengths, are nearly linearly correlated with only a small offset. Thus, the stability of $\Lambda\Lambda^6\text{He}$ ensures stability for $\Lambda\Lambda^5\text{H}$.

3 Ingredients of hypernuclear shell model

Shell-model predictions for $\Lambda\Lambda$ hypernuclei have been given recently [9] using Eq. (1) in which $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda^A Z)$ is replaced by a constant $V_{\Lambda\Lambda}$ matrix element, identified with $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda^6\text{He})$ of Eq. (2).¹ Calculations of $B_{\Lambda\Lambda}(\Lambda\Lambda^A Z)$ require then the knowledge of $\bar{B}_\Lambda(\Lambda^{A-1} Z)$ involving single- Λ hypernuclear ground-state (g.s.) binding energies plus g.s. doublet splittings $\Delta E_{\text{g.s.}}$ for $J_{\text{core}} \neq 0$. Table 1 lists $\Delta E_{\text{g.s.}}$ values relevant for the calculations reviewed here, exhibiting remarkable agreement between theory and experiment.

Table 1 Doublet splittings ΔE^{th} and ΔE^{exp} (in keV) from Refs. [25, 26], where $\Delta E_{\text{alt}}^{\text{th}}$ uses ESC04a-inspired $\Lambda - \Sigma$ coupling. Note the sensitivity of $\Delta E^{\text{th}}(^{10}_\Lambda\text{B}_{\text{g.s.}})$ to the model used for $\Lambda - \Sigma$ mixing. The $^9_\Lambda\text{Be}^*$ and $^{13}_\Lambda\text{C}^*$ excited doublets are discussed in Sect. 4.

	J_{up}^π	J_{low}^π	ΔE^{th}	$\Delta E_{\text{alt}}^{\text{th}}$	ΔE^{exp}
$^9_\Lambda\text{Be}^*$	$3/2^+$	$5/2^+$	44	49	43 ± 5
$^{10}_\Lambda\text{B}_{\text{g.s.}}$	2^-	1^-	120	34	≤ 100
$^{11}_\Lambda\text{B}_{\text{g.s.}}$	$7/2^+$	$5/2^+$	267	243	262.9 ± 0.2
$^{12}_\Lambda\text{C}_{\text{g.s.}}$	2^-	1^-	153	167	161.4 ± 0.7
$^{13}_\Lambda\text{C}^*$	$5/2^+$	$3/2^+$	31	47	—

¹ A straightforward modification for $^{10}_{\Lambda\Lambda}\text{Be}_{\text{g.s.}}(0^+)$, with a nuclear core ^8Be unstable to α emission, is discussed in Ref. [9].

4 Interpretation of ${}_{\Lambda\Lambda}^{10}\text{Be}$ and ${}_{\Lambda\Lambda}^{13}\text{B}$ emulsion events

The $B_{\Lambda\Lambda}$ values of both ${}_{\Lambda\Lambda}^{10}\text{Be}$ (17.5 ± 0.4 MeV) [2] and ${}_{\Lambda\Lambda}^{13}\text{B}$ (28.2 ± 0.7 MeV) [27] were extracted assuming that their π^- weak decay proceeds to the g.s. of the respective daughter Λ hypernuclei. This led to $\Delta B_{\Lambda\Lambda} \sim 4 - 5$ MeV, substantially higher than for ${}_{\Lambda\Lambda}^6\text{He}$ (NAGARA). However, as realized by Danysz *et al.* [1], the decay could proceed to excited states of the daughter Λ hypernucleus which deexcites then rapidly to the g.s. emitting unobserved γ radiation. This reduces the apparent $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ values by the Λ hypernuclear excitation energy involved in the π^- weak decay. Consistency with ${}_{\Lambda\Lambda}^6\text{He}$ is restored upon accepting the following weak decays:

$${}_{\Lambda\Lambda}^{10}\text{Be} \rightarrow {}_{\Lambda}^9\text{Be}^*(3/2^+, 5/2^+; 3.04 \text{ MeV}) + p + \pi^-, \quad (3)$$

$${}_{\Lambda\Lambda}^{13}\text{B} \rightarrow {}_{\Lambda}^{13}\text{C}^*(3/2^+, 5/2^+; 4.9 \text{ MeV}) + \pi^-, \quad (4)$$

with rates comparable to those for decays to ${}_{\Lambda}^9\text{Be}_{\text{g.s.}}(1/2^+)$ and ${}_{\Lambda}^{13}\text{C}_{\text{g.s.}}(1/2^+)$, respectively. The doublet splittings of ${}_{\Lambda}^9\text{Be}^*$ and ${}_{\Lambda}^{13}\text{C}^*$ are listed in Table 1.

${}_{\Lambda\Lambda}^{10}\text{Be}$ also fits the Demachi-Yanagi event observed in KEK-E373 [28], with $B_{\Lambda\Lambda} = 11.90 \pm 0.13$ MeV [8] determined from the assumed formation reaction kinematics. The approximately 6 MeV difference between this and the Danysz *et al.* [1,2] value for $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{10}\text{Be})$ is reconciled by assuming that the Demachi-Yanagi event corresponds to formation of the first excited state ${}_{\Lambda\Lambda}^{10}\text{Be}^*$,

$$\Xi^- + {}^{12}\text{C} \rightarrow {}_{\Lambda\Lambda}^{10}\text{Be}^*(2^+; \approx 3 \text{ MeV}) + t, \quad (5)$$

which decays to ${}_{\Lambda\Lambda}^{10}\text{Be}_{\text{g.s.}}$ by emitting unseen γ ray, the energy of which has to be added to the apparent $B_{\Lambda\Lambda}$ value deduced by assuming a g.s. formation. It is not clear why the formation of ${}_{\Lambda\Lambda}^{10}\text{Be}^*$ should be comparable or enhanced with respect to that of ${}_{\Lambda\Lambda}^{10}\text{Be}_{\text{g.s.}}$.

The $B_{\Lambda\Lambda}^{\text{exp}}$ values corresponding to Eqs. (3)–(5) are listed in Table 2 together with predictions made in cluster model (CM) and shell model (SM) calculations, all of which use $\Lambda\Lambda$ interactions normalized to $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) = 6.91 \pm 0.16$ MeV [8]. For ${}_{\Lambda\Lambda}^{13}\text{B}$, assuming charge symmetry, the ${}_{\Lambda}^{12}\text{B}_{\text{g.s.}}$ doublet splitting input was identified with that of ${}_{\Lambda}^{12}\text{C}_{\text{g.s.}}$ from Table 1. It is seen that both CM and SM calculations reproduce the reinterpreted $B_{\Lambda\Lambda}$ values of ${}_{\Lambda\Lambda}^{10}\text{Be}$ and ${}_{\Lambda\Lambda}^{13}\text{B}$. The SM agrees well with the Hiyama *et al.* CM calculation [10,11], and the SM calculation has no match for ${}_{\Lambda\Lambda}^{13}\text{B}$.

Table 2 Reinterpreted $B_{\Lambda\Lambda}^{\text{exp}}$ values (in MeV) and predictions based on the NAGARA event for ${}_{\Lambda\Lambda}^6\text{He}$. The error on $B_{\Lambda\Lambda}^{\text{exp}}({}_{\Lambda\Lambda}^6\text{He})$ is incorporated into the predicted values.

	$B_{\Lambda\Lambda}^{\text{exp}}$		$B_{\Lambda\Lambda}^{\text{CM}}$		$B_{\Lambda\Lambda}^{\text{SM}}$
	Eqs. (3,4)	Eq. (5)	[6]	[11]	[9]
${}_{\Lambda\Lambda}^{10}\text{Be}$	14.5 ± 0.4	14.94 ± 0.13	14.35 ± 0.19	14.74 ± 0.19	14.97 ± 0.22
${}_{\Lambda\Lambda}^{13}\text{B}$	23.3 ± 0.7		–	–	23.21 ± 0.21

5 Alternative interpretations of the ${}_{\Lambda\Lambda}^{13}\text{B}$ event

The emulsion event assigned to ${}_{\Lambda\Lambda}^{13}\text{B}$ [4, 5] has been carefully scrutinized by the KEK-E176 Collaboration [27]. Several alternative assignments were pointed out, two of which that do not require Λ hypernuclear excitation in the π^- weak decay of the $\Lambda\Lambda$ hypernuclear g.s. are listed in Table 3. Comparison with model calculations suggests that such reassignments cannot be ruled out, although a ${}_{\Lambda\Lambda}^{13}\text{B}$ assignment shows a higher degree of consistency between the $B_{\Lambda\Lambda}$ values derived from formation and from decay. In particular, the accepted formation reaction $\Xi^- + {}^{14}\text{N} \rightarrow {}_{\Lambda\Lambda}^{13}\text{B} + p + n$ was shown to occur naturally in Ξ^- capture at rest in light nuclei emulsion [5].

Table 3 Reassignments of the ${}_{\Lambda\Lambda}^{13}\text{B}$ KEK-E176 event. $B_{\Lambda\Lambda}$ values are in MeV.

	$B_{\Lambda\Lambda}^{\text{exp}}$ [27]	$B_{\Lambda\Lambda}^{\text{CM}}$ [11]	$B_{\Lambda\Lambda}^{\text{SM}}$ [9]
${}_{\Lambda\Lambda}^{11}\text{Be}$	17.53 ± 0.71	18.23 ± 0.19	18.40 ± 0.28
${}_{\Lambda\Lambda}^{12}\text{B}$	20.60 ± 0.74	–	20.85 ± 0.20

6 Interpretation of the KEK-E373 HIDA event

Table 4 Assignments suggested for the KEK-E373 HIDA event. $B_{\Lambda\Lambda}$ values are in MeV.

	$B_{\Lambda\Lambda}^{\text{exp}}$ [8]	$B_{\Lambda\Lambda}^{\text{CM}}$ [11]	$B_{\Lambda\Lambda}^{\text{SM}}$ [9]
${}_{\Lambda\Lambda}^{11}\text{Be}$	20.83 ± 1.27	18.23 ± 0.19	18.40 ± 0.28
${}_{\Lambda\Lambda}^{12}\text{Be}$	22.48 ± 1.21	–	20.72 ± 0.20

The KEK-E373 Collaboration has recently presented evidence from the HIDA event for another $\Lambda\Lambda$ hypernucleus, tentatively assigned to either ${}_{\Lambda\Lambda}^{11}\text{Be}$ or to ${}_{\Lambda\Lambda}^{12}\text{Be}$ [8]. The associated $B_{\Lambda\Lambda}^{\text{exp}}$ values, together with model predictions, are listed in Table 4. We note that since no experimental data exist on ${}_{\Lambda}^{11}\text{Be}$, the required input for evaluating $B_{\Lambda\Lambda}^{\text{SM}}({}_{\Lambda\Lambda}^{12}\text{Be})$ was derived within the SM approach [9]. It is clear from the table that neither of the proposed assignments is favorable, although the relatively large experimental uncertainties do not completely rule out either of these.

7 Conclusion

It was shown how the three acceptable $\Lambda\Lambda$ emulsion events, corresponding to ${}_{\Lambda\Lambda}^6\text{He}$, ${}_{\Lambda\Lambda}^{10}\text{Be}$ and ${}_{\Lambda\Lambda}^{13}\text{B}$, can be made consistent with each other, in good agreement with CM and with SM calculations of $B_{\Lambda\Lambda}$. Other possible assignments

for the KEK-E176 ${}^{13}_{\Lambda\Lambda}\text{B}$ event were discussed, and the assignments proposed for the recently reported HIDA event were found unfavorable. It was pointed out that simple shell-model estimates, making use of Λ -hypernuclear spectroscopic data and analysis, are sufficient for discussing the world data of $\Lambda\Lambda$ hypernuclear events. A relatively weak $\Lambda\Lambda$ interaction, with $(1s_\Lambda)^2$ matrix element of magnitude $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) = 0.67 \pm 0.17$ MeV, describes well the data in the observationally accessible mass range $6 \leq A \leq 13$. Comparably weak $\Lambda\Lambda$ interactions are obtained also in recent theoretical models, in Nijmegen extended soft-core (ESC) models [24,29] and in lowest order χEFT [30]. Less well determined is the $\Lambda\Lambda$ coupling to the slightly higher ΞN channel, with appreciable model dependence in ESC models [24,29]. The observation of $A = 5$ $\Lambda\Lambda$ hypernuclei would add valuable new information on this issue.

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